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77. Proposed by CHAS. C. CROSS, Laytonsville, Md.

A line is drawn perpendicular to BC , of the triangle ABC , whose sides are $BC=a$, $CA=b$, and $AB=c$, through A to D , a distance d , (d being equal to or greater than $a+b$); from D a line is drawn to E , a distance e , (e being equal to or greater than $a+b+c$) on BC extended. Required the area of the ellipse which is isogonal conjugate to the straight line DE with respect to the triangle ABC .

I. Solution by G. B. M. ZERR, A. M., Ph. D., President of Russell College, Lebanon, Va.

Using trilinear coördinates and letting F be the point where $AD=d$ cuts BC , we get $DF=(d-b\sin C)$, $EF=\sqrt{e^2-(d-b\sin C)^2}=f$.

\therefore The coördinates of D , E are respectively,

$$\begin{aligned} &\{-(d-b\sin C), \operatorname{ecos} C, \operatorname{ecos} B\} \text{ and} \\ &\{0, -(f-b\cos C)\sin C, -(f+c\cos B)\sin B\}. \end{aligned}$$

$$\text{Let } l=e\{(f-b\cos C)\sin C\cos C-(f+c\cos B)\sin B\cos B\},$$

$$m=-(d-b\sin C)(f+c\cos B)\sin B, \text{ and}$$

$$n=(d-b\sin C)(f-b\cos C)\sin C.$$

Then $l\alpha+m\beta+n\gamma=0$, is the equation to DE , and $l\beta\gamma+m\gamma\alpha+n\alpha\beta=0$, is the equation to the ellipse isogonal conjugate to DE .

Let B be the origin, BC , BA the axes of (x, y) .

Then $\alpha=y\sin B$, $\gamma=x\sin B$, $\beta=(a\cos B-a\alpha-c\gamma)/b$.

$\therefore \beta=\sin B(ac-ay-cx)/b$.

Substituting these values of α , β , γ the equation to the ellipse becomes,

$$clx^2+any^2+(al+cn-bm)xy-aclx-acny=0.$$

Let $\Delta = \frac{a^2c^2\ln(2-cn-ae)}{4acln-(al+cn-bm)^2}$ be the discriminant of the ellipse.

The two values of z in the equation,

$$z^2 + \frac{16(cl+an)\Delta}{\{4acln-(al+cn-bm)^2\}^2} z - \frac{64\Delta^2}{\{4acln-(al+cn-bm)^2\}^3} = 0,$$

give the values of the squares of the semi-axes.

\therefore Area of ellipse

$$= \frac{8\pi\Delta}{\{4acln-(al+cn-bm)^2\}^{\frac{3}{2}}} = \frac{8\pi a^2c^2\ln(2-cn-al)}{\{4acln-(al+cn-bm)^2\}^{\frac{3}{2}}}.$$

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

The coördinates of D are $(d, -\beta_1, -\gamma_1)$, and of E , $(0, \beta_2, -\gamma_2)$, and we then have,

$$e^2 = (abc)/(4\Delta^2) \{a(\beta_1 + \beta_2)(\gamma_2 - \gamma_1) + bd(\gamma_1 - \gamma_2) + cd(\beta_1 + \beta_2)\} \dots\dots\dots (1).$$

The equation to the perpendicular to BC through A is

$$\gamma \cos C - a \cos A = 0 \dots\dots\dots (2),$$

and this, passing through D , gives

$$\gamma_1 \cos C + d \cos A = 0 \dots\dots\dots (3).$$

We have the constant relation

$$a\alpha + b\beta + c\gamma = 2\Delta \dots\dots\dots (4),$$

and this being satisfied by the coördinates of D and E ,

$$ad - b\beta_1 - c\gamma_1 = 2\Delta \dots\dots\dots (5).$$

$$b\beta_2 + c\gamma_2 = 2\Delta \dots\dots\dots (6).$$

The equation to DE is

$$\alpha(\beta_1\gamma_2 + \beta_2\gamma_1) + \beta d\gamma_2 + \gamma d\beta_2 = 0 \dots\dots\dots (7),$$

the isogonal conjugate of which is

$$\beta\gamma(\beta_1\gamma_2 + \beta_2\gamma_1) + \alpha\gamma d\gamma_2 + \alpha\beta d\beta_2 = 0 \dots\dots\dots (8),$$

which by the problem is an ellipse.

The area of (8) is expressed by

$$K = 2\pi \Delta abc \left\{ \begin{vmatrix} 0, & \frac{1}{2}d\beta_2, & \frac{1}{2}d\gamma_2 \\ \frac{1}{2}d\beta_2, & 0, & \frac{1}{2}(\beta_1\gamma_2 + \beta_2\gamma_1) \\ \frac{1}{2}d\gamma_2, & \frac{1}{2}(\beta_1\gamma_2 + \beta_2\gamma_1), & 0 \end{vmatrix} \div \right. \\ \left. \cdot \begin{vmatrix} 0, & \frac{1}{2}d\beta_2, & \frac{1}{2}d\gamma_2, & -a \\ \frac{1}{2}d\beta_2, & 0, & \frac{1}{2}(\beta_1\gamma_2 + \beta_2\gamma_1), & -b \\ \frac{1}{2}d\gamma_2, & \frac{1}{2}(\beta_1\gamma_2 + \beta_2\gamma_1), & 0, & -c \\ a, & b, & c, & 0 \end{vmatrix}^{\frac{3}{2}} \right\} \dots\dots (9).$$

β_1, γ_1 are determined by (3) and (5), and then β_2 and γ_2 from (1) and (6), giving K in terms of d and elements of the triangle of reference.

It is not obvious how much of a reduction (9) admits, and I have not attempted any.